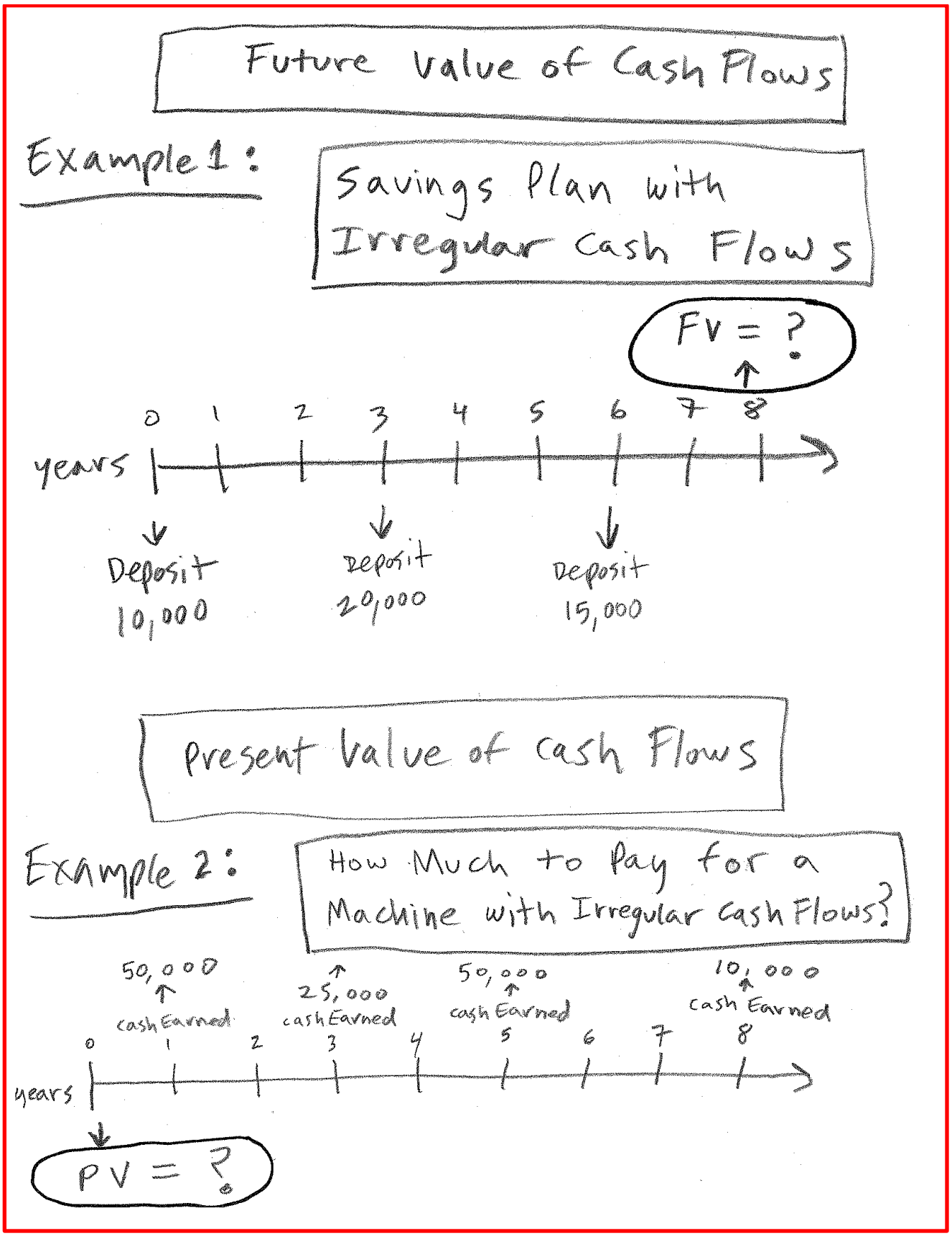
Excel & Business Math
Video/Class Project #45
Cash Flow Analysis for Annuities: Savings Plans, Asset Valuation, Retirement Plans and Mortgage Loan. FV, PV and PMT.

Topics

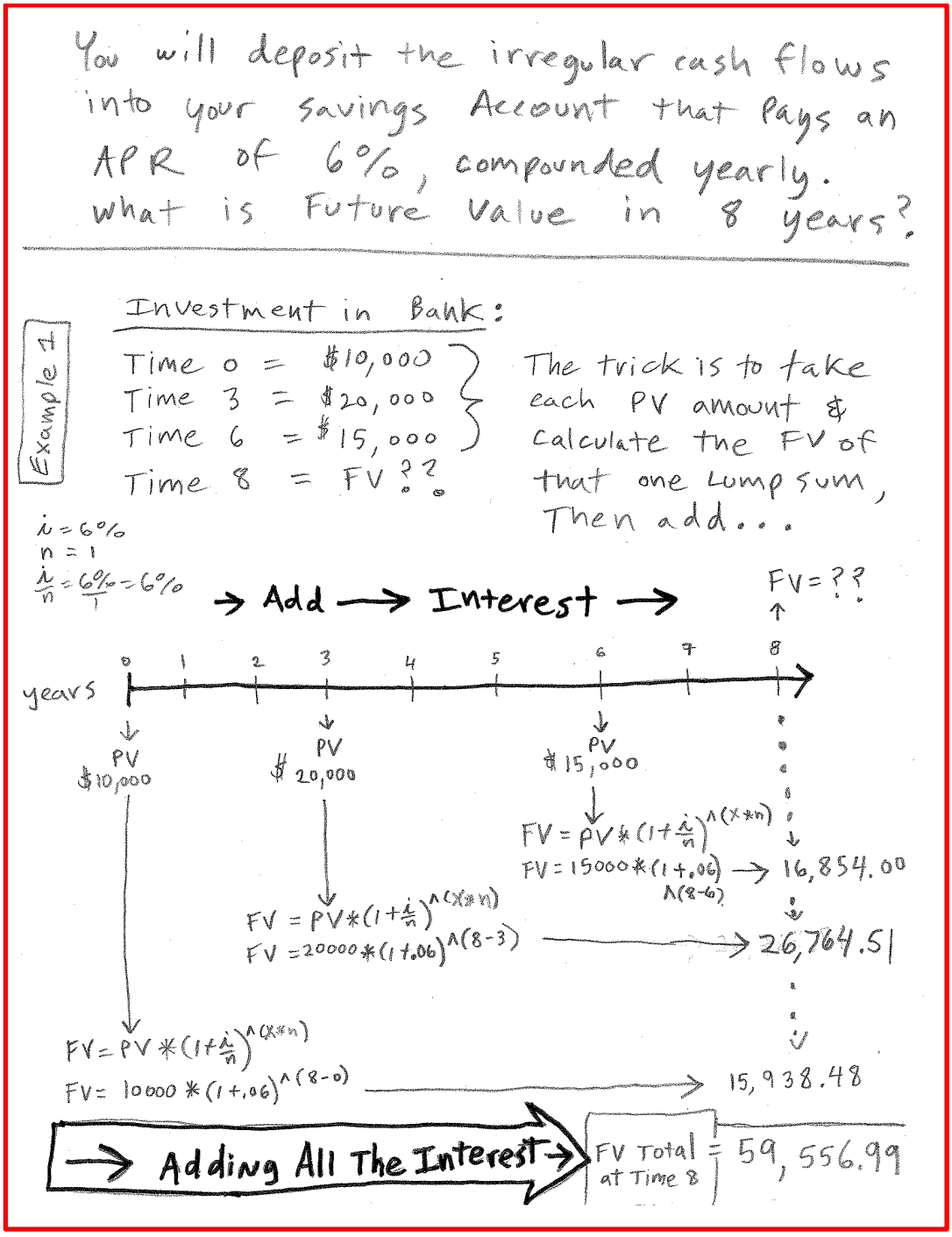
1) Cash Flow Pattern Diagram for Future Value and Present Value of Irregular Cash Flows

- Example 1: Below is a picture of Cash Flows from a Savings Plan. What is Future Value of Savings Plan?
- Example 2: Below is a picture of Cash Flows a New Machine can Earn. How Much should we pay for such a machine?



2) Future Value Calculation of Savings Plan with Irregular Cash Flows, Hand Drawings & Diagram.

- If we are depositing amounts of money into our bank account at the end of each few years (not each year), we can make a Future Value Calculation on each individual amount to calculate what each Lump Sum's Future Value is for each deposited amount, and then add them to get the Total Future Amount in our Bank Account.
- Define "Irregular Cash Flows"
    - The Cash Flow Amounts are called "Irregular" because the amounts are not the same each period and the time between each cash flow is not the same. This is to distinguish the Cash Flows from Regular or Periodic Cash flows, which are equal in amount each period and the time between each amount in equal.
- The below picture illustrates how each deposited amount is an individual Present Value Lump Sum Amount that we must use to calculate its Future Value amount. When we add the Future Value amounts we get the amount in the bank account at the end of all the periods.



3) Excel Example 1: Calculate Future Value of Savings Plan with Irregular Cash Flows.

Example 1:

Future Value Calculation of Savings Plan with Irregular Cash Flows.

You will deposit the Cash Flows (listed below) into your Savings Account that pays 6%, compounded yearly
What is the Future Value after 8 years?

Variable	Math Variable	Excel Variable	Number
Annual Rate (APR)	i		0.06
Compounding Periods per year	n		1
Years	x		8
Period Rate	i/n	rate	0.06

Formula in cell E16:	Formula in cell F16:	Formula in cell G16:
=$D$10-B16	=FV($D$11,E16,,-D16)	=D16*(1+$D$11)^E16

Years	Date	Irregular Cash Flows Into Bank, each is a PV amount	Total Number of Periods	FV (each calculation is a Lump Sum Calculation)	FV Check
0	12/31/2017	$10,000.00	8	$15,938.48	$15,938.48
1	12/31/2018		7	$0.00	$0.00
2	12/31/2019		6	$0.00	$0.00
3	12/31/2020	$20,000.00	5	$26,764.51	$26,764.51
4	12/31/2021		4	$0.00	$0.00
5	12/31/2022		3	$0.00	$0.00
6	12/31/2023	$15,000.00	2	$16,854.00	$16,854.00
7	12/31/2024		1	$0.00	$0.00
8	12/31/2025		0	$0.00	$0.00
			Total FV	$59,556.99	$59,556.99

Formula in cell F26:	Formula in cell G26:
=SUM(F16:F24)	=SUM(G16:G24)

4) Present Value Calculation of Irregular Future Cash Flows. Asset Valuation Calculation. Hand Drawings & Diagram.

- In this example, a company has estimated that a new machine they are considering buying will generate future profit (Cash Flows) for the company, as see here:

Years	Date	Irregular Cash Flows the Machine will Generate, each is a FV amount
0	12/31/2017	
1	12/31/2018	$50,000.00
2	12/31/2019	
3	12/31/2020	$25,000.00
4	12/31/2021	
5	12/31/2022	$50,000.00
6	12/31/2023	
7	12/31/2024	
8	12/31/2025	$10,000.00

- If the Company has to earn an Annual Interest Rate or Rate of Return of 17% once they buy the machine, we can estimate what the new machine should cost by calculating the Present Value Amount for each Lump Sum Future Value Amount. In Essence, for each FV Lump Sum Cash Flow, we can "discount it back to Time 0", taking all of the interest that we expect to earn, and then add the Present Value amounts to estimate that maximum we should pay for the machine. This example is illustrated in the diagram on the next page.
- Terms:
    - Asset = Something we own or control that brings benefit to the company, like cash.
    - Time 0 (Time Zero) is the time when we buy the asset.
    - "discount back to time 0" = remove all the assumed interest from a FV Amount to get the PV Amount at Time 0.
    - Asset Valuation Calculation = discount all future cash flows that asset will generate back to Time 0 so you can estimate the maximum amount that should be paid for the asset.
    - Future Cash Flows = Each Cash Flow in the future is an estimate at a point in time that considers all the cash flows in and all the cash flows out. It is an estimate of the cash flow profit from the asset.

Example 2

Cash Flows Generated by Asset:

Time 1 = $50,000
Time 3 = $25,000
Time 5 = $50,000
Time 8 = $10,000

The trick is to take each FV amount & calculate the PV of that one Lump Sum, Then add . . .

$\frac{i}{n} = 17\%$

← Subtracting Interest ←

PV = ??

PV total at Time 0

FV $50,000
$PV = \frac{FV}{(1+\frac{i}{n})^{(x*n)}}$
$\frac{50000}{(1+.17)^1}$ ← 42,735.04

FV $25,000
$PV = \frac{FV}{(1+\frac{i}{n})^{(x*n)}}$
$\frac{25000}{(1+.17)^3}$ ← 15,609.26

FV $50,000
$PV = \frac{FV}{(1+\frac{i}{n})^{(x*n)}}$
$\frac{50000}{(1+.17)^5}$ ← 22,805.56

FV $10,000
$PV = \frac{FV}{(1+\frac{i}{n})^{(x*n)}}$
$\frac{10000}{(1+.17)^8}$ ← 2,847.82

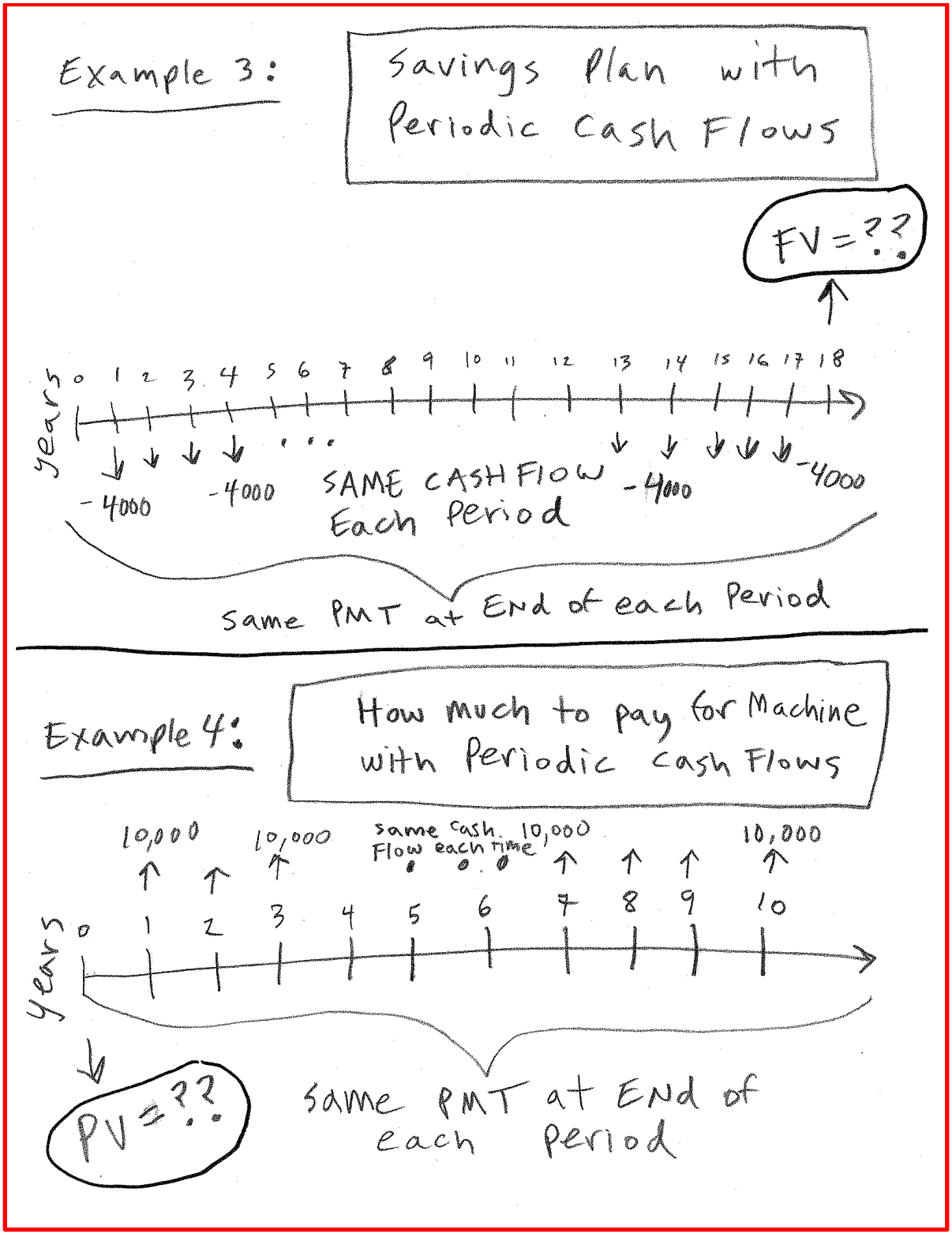
SUBTRACTING ALL THE INTEREST

83,997.69 = PV Total at Time 0 — Max to Pay For Asset

5) Excel Example 2: Calculate Present Value of Irregular Future Cash Flows to determine Asset Valuation.

	A	B	C	D	E	F	G	H
1	**Example 2:**							
2	**Present Value Calculation of Irregular Future Cash Flows. Asset Valuation Calculation.**							
3								
4	How much should you pay for a Machine for your business if it is estimated that it will generate the Cash Flows listed below							
5	Your business requires that the machine helps the business earn a 17% return (like interest or investment return).							
6								
7	Variable	Math Variable	Excel Variable	Number				
8	Required Rate of Return Annual Rate (APR) or Discount Rate	i		0.17				
9	Compounding Periods per year	n		1				
10	Years	x		8				
11	Period Rate	i/n	rate	0.17				
12								
13					Formula in cell E16:	Formula in cell F16:		Formula in cell H16:
14					=PV($D$11,B16,,D16)	=D16/(1+$D$11)^B16		=FV($D$11,B16,,E16)
15		Years	Date	Irregular Cash Flows the Machine will Generate, each is a FV amount	PV (each calculation is a Lump Sum Calculation)	PV Check		Prove using FV
16		0	12/31/2017		$0.00	$0.00		$0.00
17		1	12/31/2018	$50,000.00	-$42,735.04	$42,735.04		$50,000.00
18		2	12/31/2019		$0.00	$0.00		$0.00
19		3	12/31/2020	$25,000.00	-$15,609.26	$15,609.26		$25,000.00
20		4	12/31/2021		$0.00	$0.00		$0.00
21		5	12/31/2022	$50,000.00	-$22,805.56	$22,805.56		$50,000.00
22		6	12/31/2023		$0.00	$0.00		$0.00
23		7	12/31/2024		$0.00	$0.00		$0.00
24		8	12/31/2025	$10,000.00	-$2,847.82	$2,847.82		$10,000.00
25								
26				Total PV	-$83,997.69	$83,997.69	<<== Maximum to pay for Asset	
27								
28					Formula in cell E26:	Formula in cell F26:		
29					=SUM(E16:E24)	=SUM(F16:F24)		
30								
31				If the Machine Cost 72,500, wopuld you buy it?		Yes!		
32				If the Machine Cost 85,500, wopuld you buy it?		No!		

6) Cash Flow Pattern Diagram for Future Value and Present Value of Periodic Cash Flows



7) If We have Regular or Periodic Cash Flows, rather than Irregular Cash Flows, our Calculations for a FV or PV Amount are Much Easier!!!

- Regular Cash Flows are defined as Cash Flows that are the same amount each period and the time between each cash flow is the same. When you have a cash flow pattern like this, it is called an ***Annuity***. Regular Cash Flows are also called Periodic Cash Flows or Periodic Payments.

8) Define Terms for an Annuity

Define Annuity

An Annuity is a financial debt or investment vehicle that contains Periodic Cash Flows that meet this definition:

1) Equal amount of Cash Flow each period
2) Time between each Cash Flow is the equal

Two Types of Annuities:

End Annuities: Cash Flows ocurr at the end of each period: Also called "Ordinary" Annuities

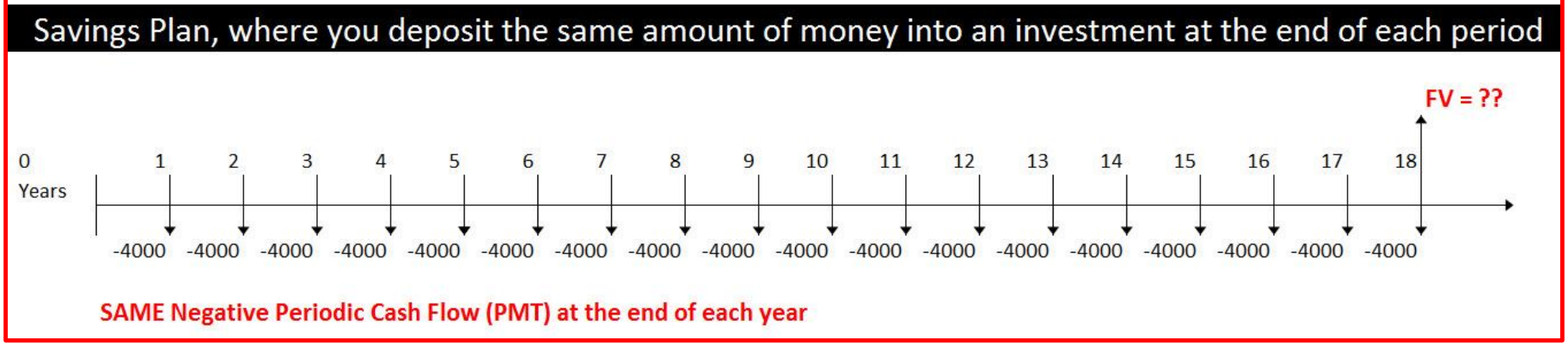
Begin Annuities: Cash Flows ocurr at the beginning of each period: Also called "Due" Annuities

** In this class we will study End Annuities.

Examples of End Annuities:

Savings Plan, where you deposit the same amount of money into an investment at the end of each period	Example 3
Buying an Asset that will generate the same amount of cash flow at the end of each period	Example 4
Receiving a Retirement Check for the same amount at the end of each month	Example 5
Home Loans or Car Loans, where the monthly payment is the same at the end of each month	Example 6

Picture of Savings Plan Annuity Cash Flow Pattern:

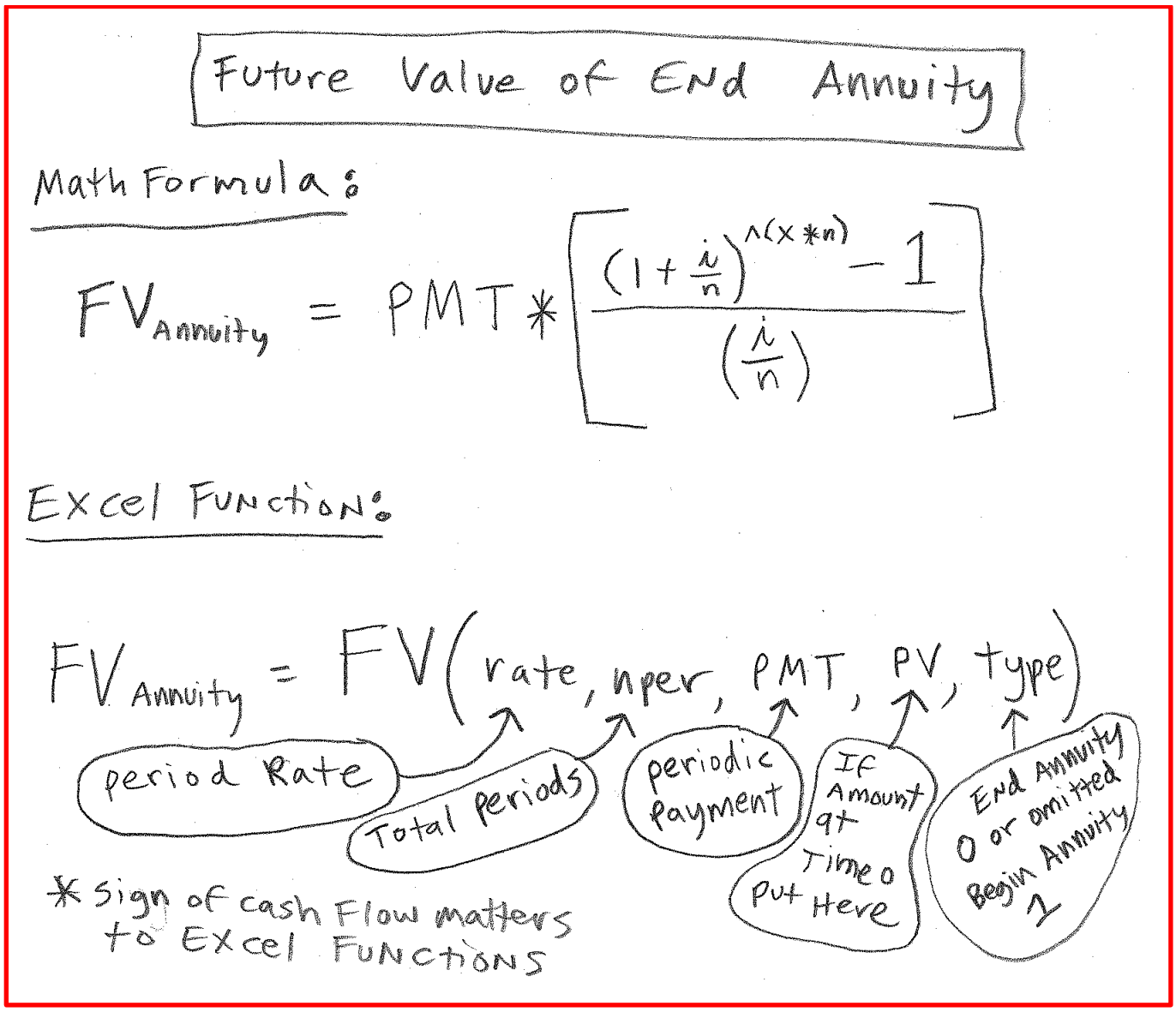


9) Terms & Variables for Annuity Calculations

Financial Variable Name	Math Symbol	Excel Function Argument
Annual Rate (APR), Discount Rate, Rate of Return	i	
Compounding Periods per year	n	
Years	x	
Period Rate	i/n	rate
Total Number of Compounding Periods	x*n	nper
Present Value	PV	pv
Future Value	FV	fv
Periodic Cash Flow / Periodic Payment	PMT	pmt

** PV, FV and pmt arguments in Excel must have the correct Cash Flow Direction.

10) Formulas for Calculating Future Value of an End Annuity

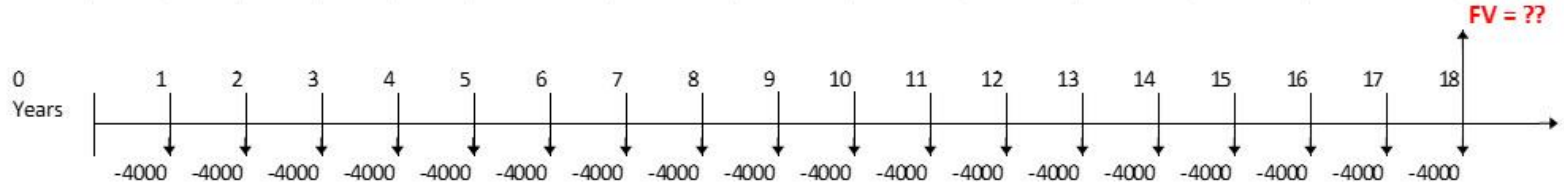


11) Excel Example 3: Calculate Future Value for Savings Plan with Periodic Cash Flow in an End Annuity.

Example 3:

Savings Plan with Periodic Cash Flow, Calculate Future Value

Savings Plan, where you deposit the same amount of money into an investment at the end of each period



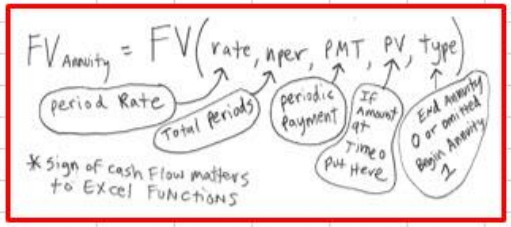
SAME Negative Periodic Cash Flow (PMT) at the end of each year

At the end of each year you plan to deposit $4,000 into a Savings Plan that pays 5%, compounded yearly.
What is the Future Value of this Savings Plan in 18 years?

Does it meet the requirements for an End Annuity?

1) Equal amount of Cash Flow each period	Yes ✓
2) Time between each Cash Flow is the equal	Yes ✓

Financial Variable Name	Math	Excel	Value	
Annual Rate	i		5%	
Compounding Periods per year	n		1	
Years	x		18	
Periodic Cash Flow / Payment	PMT	pmt	4000	
Direction of Cash Flow?			Negative	
Present Value	PV	pv	NA	
Period Rate	i/n	rate	0.05	=G21/G22
Total # of Compounding Periods	x*n	nper	18	=G23*G22
Future Value	FV	fv	$112,529.54	=FV(G27,G28,-G24)
Total Amount Deposited = PMT * x*n =			$72,000.00	=G24*G23
Total Interest = FV - Total Amount Deposited =			$40,529.54	=G29-G30
Check: Future Value:			$112,529.54	=G24*(((1+G27)^G28-1)/G27)



FV =PMT*((1+i/n)^(x*n)-1)/(i/n)

Check Long Hand:

Cash Flow	Year	FV	
0	0	$0.00	
-4000	1	$9,168.07	=FV($G$21,$G$23-M20,,L20)
-4000	2	$8,731.50	
-4000	3	$8,315.71	
-4000	4	$7,919.73	
-4000	5	$7,542.60	
-4000	6	$7,183.43	
-4000	7	$6,841.36	
-4000	8	$6,515.58	
-4000	9	$6,205.31	
-4000	10	$5,909.82	
-4000	11	$5,628.40	
-4000	12	$5,360.38	
-4000	13	$5,105.13	
-4000	14	$4,862.03	
-4000	15	$4,630.50	
-4000	16	$4,410.00	
-4000	17	$4,200.00	
-4000	18	$4,000.00	
Total Future Value:		$112,529.54	=SUM(N20:N37)

12) Formulas for Calculating Present Value of an End Annuity

Present Value of End Annuity

Math Formula:

minus

$$PV_{Annuity} = PMT * \left[\frac{1 - \left(1 + \frac{i}{n}\right)^{-(x*n)}}{\left(\frac{i}{n}\right)} \right]$$

Excel Function:

$$PV_{Annuity} = PV(rate, nper, PMT, FV, type)$$

- rate: Period Rate
- nper: Total # periods
- PMT: periodic payment
- FV: If amount at End of Last period, put Here
- type: End annuity 0 or omitted, Begin Annuity 1

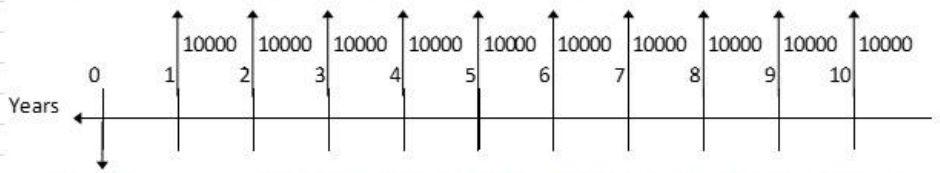
* Sign of cash Flow Matters in Excel Functions

13) Excel Example 4: Calculate Present Value of Future Periodic Cash Flows (End Annuity) to determine Asset Valuation.

Example 4:

Asset Valuation with Periodic Cash Flow, Calculate Present Value

Buying an Asset that will generate the same amount of cash flow at the end of each period



PV = ?? **SAME Positve Periodic Cash Flow (PMT) at the end of each year**

How much should you pay for a Machine for your business if it is estimated that it will generate $10,000 in Cash Flows at the end of each year for 10 years.
Your business requires that the machine helps the business earn a 17% return (like interest or investment return).
What is the maximum that we should pay for the Machine?

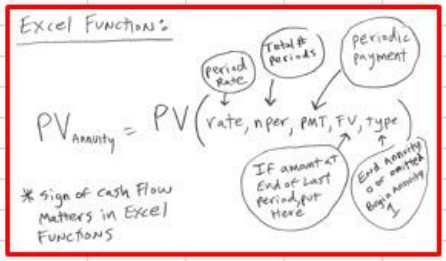
Does it meet the requirements for an End Annuity?

1) Equal amount of Cash Flow each period	Yes ✓
2) Time between each Cash Flow is the equal	Yes ✓

Financial Variable Name	Math	Excel	Value	
Annual Rate / Required Return / Discount Rate	i		17%	
Compounding Periods per year	n		1	
Years	x		10	
Periodic Cash Flow / Payment	PMT	pmt	10000	
Direction of Cash Flow?			Positive	
Future Value	FV	fv	NA	
Period Rate	i/n	rate	0.17	=H22/H23
Total # of Compounding Periods	x*n	nper	10	=H24*H23
Present Value = Max To Pay for Asset =	PV	pv	($46,586.04)	=PV(H28,H29,H25)
If the Machine Cost 45,500, would you buy it?				Yes!
If the Machine Cost 52,500, would you buy it?				No!

Check Long Hand:

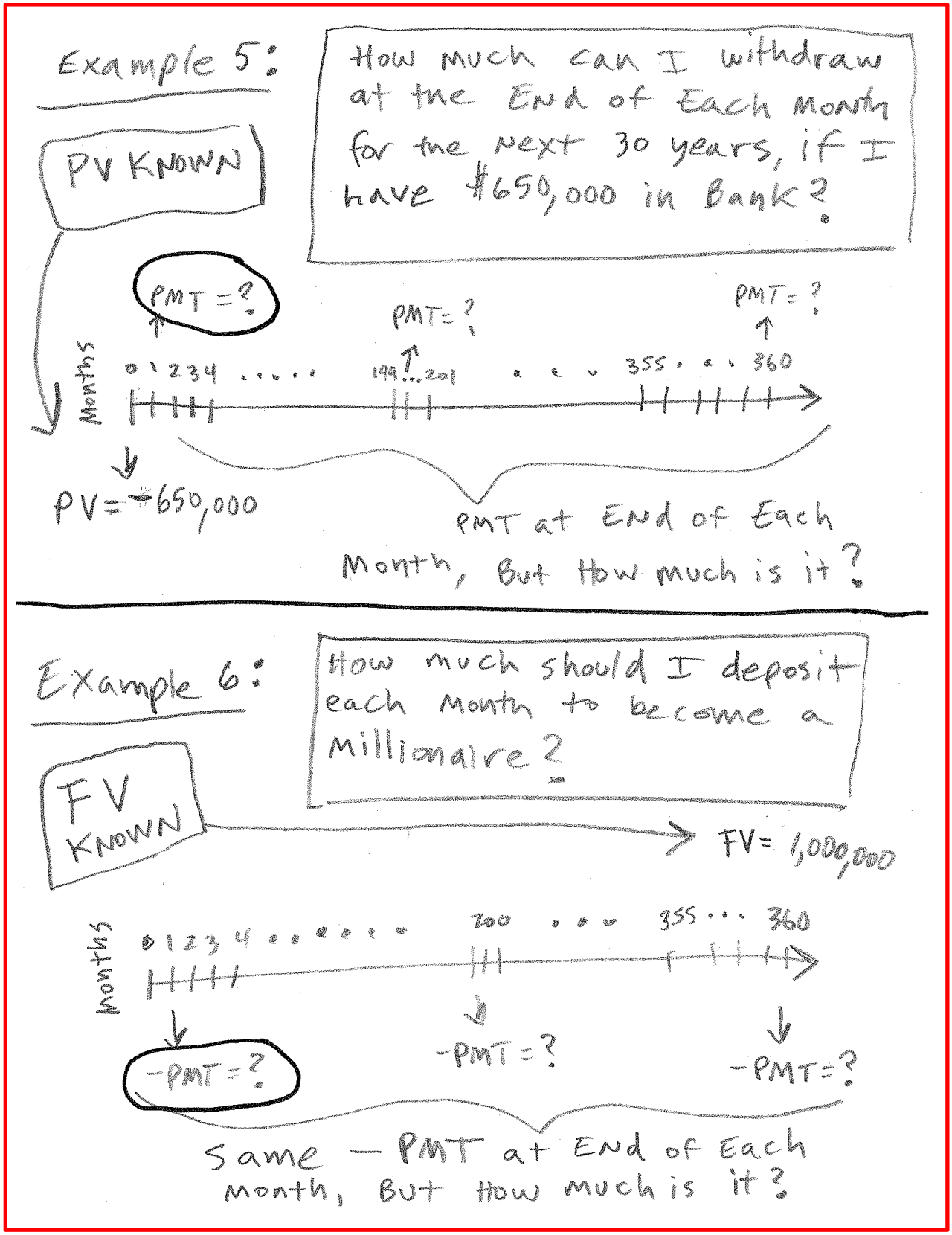
Cash Flow	Year	PV	
0	0	$0.00	
10000	1	($8,547.01)	=PV($H$28,M22,,L22)
10000	2	($7,305.14)	
10000	3	($6,243.71)	
10000	4	($5,336.50)	
10000	5	($4,561.11)	
10000	6	($3,898.39)	
10000	7	($3,331.95)	
10000	8	($2,847.82)	
10000	9	($2,434.04)	
10000	10	($2,080.37)	
Total Present Value:		($46,586.04)	=SUM(N21:N31)



Check: Present Value: 46586.03628 =H25*((1-(1+H28)^-H29)/H28)

PV =PMT*((1+i/n)^-(x*n)-1)/(i/n)

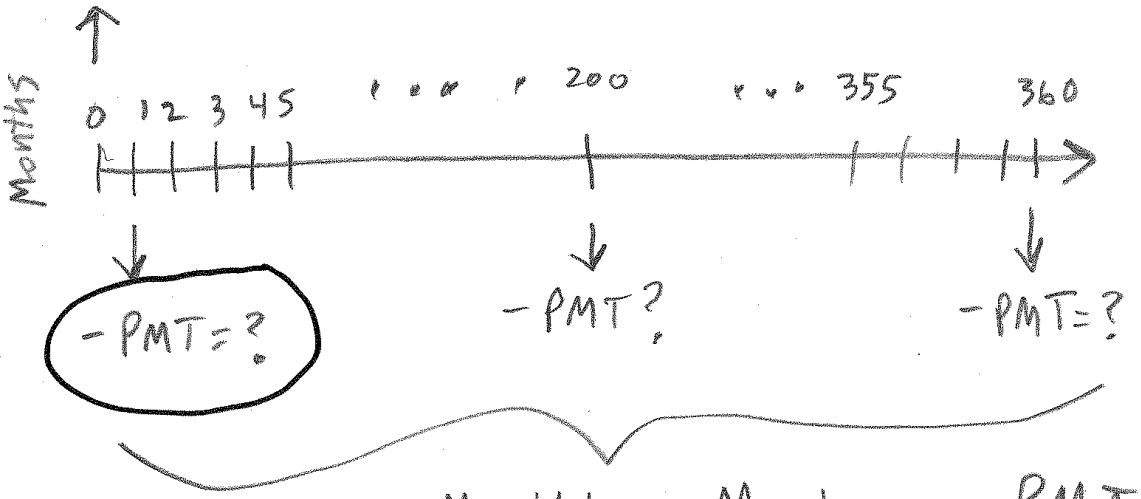
14) Cash Flow Pattern Diagram for Future Value and Present Value Annuities when we are Solving for the PMT (Periodic Cash Flow) Amount



Example 7:

If I borrow $500,000 to buy a House, what is My Monthly PMT?

PV = 500,000 ← PV KNOWN



your Monthly Mortgage PMT at the end of each month. How much is it?

15) Formulas for Calculating the PMT Value in an End Annuity When Present Value is Known

Solve for PMT in Present Value End Annuity

Math Formulas:

$$PMT_{PV_{Annuity}} = \frac{PV_{Annuity}}{\left[\frac{1-(1+\frac{i}{n})^{\wedge -(x*n)}}{(\frac{i}{n})}\right]} \leftarrow \text{minus}$$

Excel Function:

$$PMT_{PV_{Annuity}} = PMT(rate, nper, PV, FV, type)$$

- rate: period rate
- nper: Total # periods
- PV: Amount at Time 0
- FV: Amount at End of last period
- type: End Annuity 0 or omitted; Begin Annuity 1

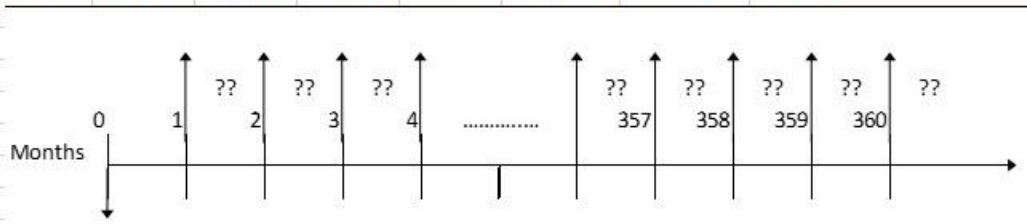
*Sign of cash Flow matters for Excel Functions

16) Excel Example 5: Calculate PMT When Present Value Amount is Known. How Much Can We Withdraw at End of Each Month for Next 30 Years?

Example 5:

Retirement Plan with $650,000 in Bank, What is Monthly Withdrawal We Can Make for Next 30 Years?

Receiving a Retirement Check for the same amount at the end of each month



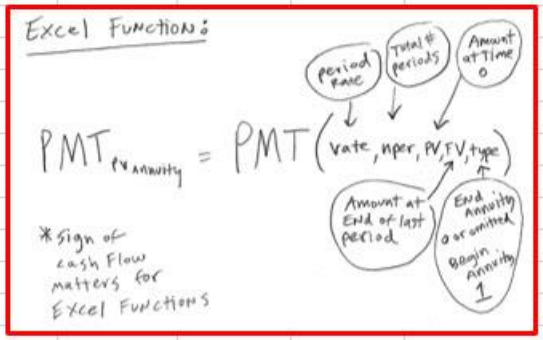
PV = -$650,000 **SAME Positve Periodic Cash Flow (PMT) at the end of each Month**

How much can I withdraw at the end of each month for the next 30 years
If I have $650,000 in the bank at 3.75%, compounded Monthly?

Does it meet the requirements for an End Annuity?

1) Equal amount of Cash Flow each period		Yes ✓
2) Time between each Cash Flow is the equal		Yes ✓

Financial Variable Name	Math	Excel	Value	
Annual Rate	i		3.75%	
Compounding Periods per year	n		12	
Years	x		30	
Present Value	PV	pv	650000	
Direction of Cash Flow?			Negative	
Future Value	FV	fv	NA	
Period Rate	i/n	rate	0.003125	=H22/H23
Total # of Compounding Periods	x*n	nper	360	=H24*H23
Periodic Cash Flow / Payment	PMT	pmt	$3,010.25	=ROUND(PMT(H28,H29,-H25),2)
	PMT Check:		$3,010.25	=H25/((1-(1+H28)^-H29)/H28)
	Total Amount Withdrawn		$1,083,690.00	=H30*H29
	Total Interest		$433,690.00	=H33-H25



PMT =PV/((1-(1+i/n)^-(x*n))/(i/n))

17) Formulas for Calculating the PMT Value in an End Annuity When Future Value is Known

Solve for PMT in Future Value End Annuity

Math Formula:

$$PMT_{FV_{Annuity}} = \frac{FV_{Annuity}}{\left[\frac{\left(1+\frac{i}{n}\right)^{\wedge(x*n)} - 1}{\left(\frac{i}{n}\right)}\right]}$$

Excel Function:

$$PMT_{FV_{Annuity}} = PMT(rate, nper, pv, FV, type)$$

- rate: period rate
- nper: Total # periods
- pv: If amount at Time 0 put Here
- FV: Future Value Amount that periodic Payments & PV amounts grow to
- type: End Annuity 0 or omitted Begin Annuity 1

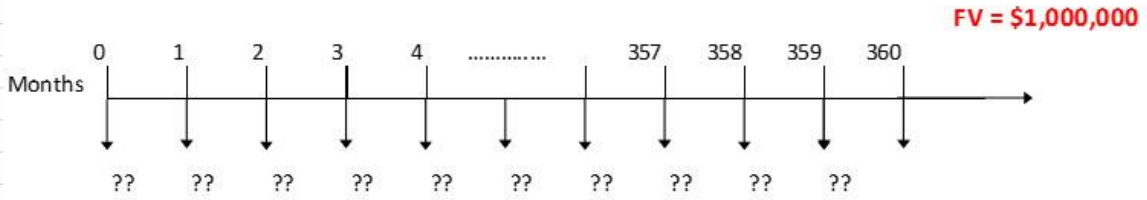
*Sign of Cash Flow matters in Excel Functions

18) Excel Example 6: Calculate PMT When Future Value Amount is Known. How Much Should I Deposit at the End of Each Month to Become Millionaire?

Example 6:

How Much Should I Deposit Each Month for Next 30 Years to Become a Millionaire?

How Much to Deposit Each Month to Become Millionaire? Periodic PMT is an Annuity

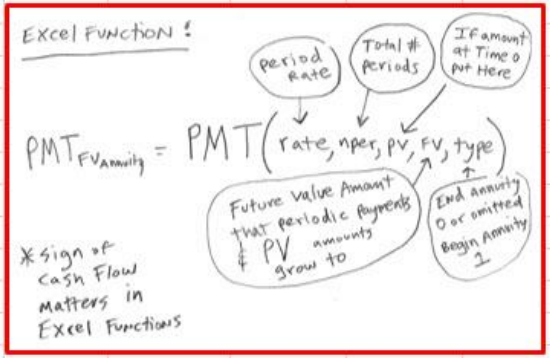


How Much Do I Need To Deposit at the End of Each Month to Become a Millionaire?
We will assume we can invest in Stocks in an Index Fund that will earn 10%, compiounded Monthly

Does it meet the requirements for an End Annuity?

1) Equal amount of Cash Flow each period	Yes ✓
2) Time between each Cash Flow is the equal	Yes ✓

Financial Variable Name	Math	Excel	Value	
Annual Rate / Required Return / Discount Rate	i		10.00%	
Compounding Periods per year	n		12	
Years	x		30	
Present Value	PV	pv	NA	
Future Value	FV	fv	1000000	
Direction of Cash Flow?			Positive	
Period Rate	i/n	rate	0.008333333	=H22/H23
Total # of Compounding Periods	x*n	nper	360	=H24*H23
Periodic Cash Flow / Payment	PMT	pmt	($442.38)	=ROUND(PMT(H28,H29,,H26),2)
	PMT Check:		$442.38	=H26/(((1+H28)^H29-1)/H28)
	Total Amount Deposited		($159,256.80)	=H30*H29
	Total Interest		$840,743.20	=H26+H33



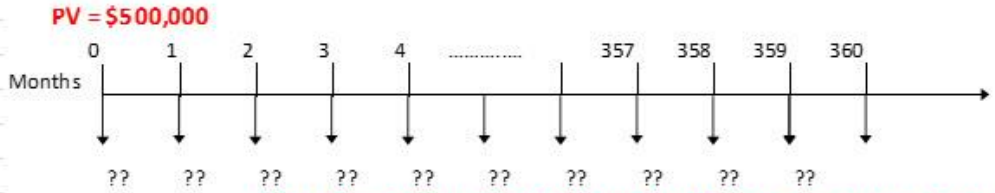
PMT =FV/(((1+i/n)^(x*n)-1)/(i/n))

19) Excel Example 7: Calculate PMT For a Home Mortgage Loan, Where Loan is Positive Present Value Amount.

Example 7:

How Much is My Monthly House PMT, If I borrow $500,000 for 30 Years?

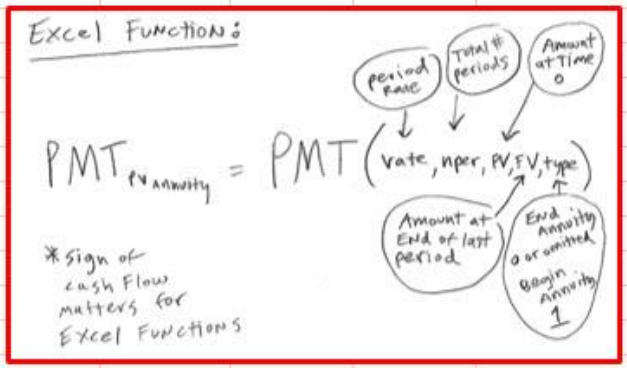
How Much to Deposit Each Month to Become Millionaire? Periodic PMT is an Annuity



Does it meet the requirements for an End Annuity?

1) Equal amount of Cash Flow each period	Yes ✓
2) Time between each Cash Flow is the equal	Yes ✓

Financial Variable Name	Math	Excel	Value	
Annual Rate / Required Return / Discount Rate	i		4.25%	
Compounding Periods per year	n		12	
Years	x		30	
Present Value	PV	pv	500000	
Direction of Cash Flow?			Positive	
Future Value	FV	fv	NA	
Period Rate	i/n	rate	0.003541667	=H22/H23
Total # of Compounding Periods	x*n	nper	360	=H24*H23
Periodic Cash Flow / Payment	PMT	pmt	($2,459.70)	=PMT(H28,H29,H25)
	PMT Check:		$2,459.70	=H25/((1-(1+H28)^-H29)/H28)



PMT =PV/((1-(1+i/n)^-(x*n))/(i/n))

20) Summary of Annuity Formulas:

FV & PMT_{FV} for End Annuity

Excel Functions:

FV (Savings Plan) =FV(rate , nper , -PMT)

PMT (Savings Plan) =PMT(rate , nper , , FV)

** Skip PV arguments (put 2 commas)

** If you put –PV in, it just means you had some $ in bank to start...

Math Formulas typed in Excel:

FV =PMT*((1+i/n)^(x*n)-1)/(i/n)

PMT =FV/(((1+i/n)^(x*n)-1)/(i/n))

PV & PMT_{PV} for End Annuity

Excel Functions:

PMT (Borrower Loan) =PMT(rate , nper , PV)

** PV positive because bank lends it to you

PV (Asset Valuation) =PV(rate , nper , PMT)

** PMT positive because cash come into business

** Resulting PV negative because that is max you should pay for asset

Math Formulas typed in Excel:

PMT =PV/((1-(1+i/n)^-(x*n))/(i/n))

PV =PMT*((1-(1+i/n)^-(x*n))/(i/n))